

Draft Tri-State New England (TSNE) Mathematics Test Specifications

These Draft Tri-State New England (TSNE) Mathematics Test Specifications are in development and are scheduled for completion by January 2004.

A Vision of a Comprehensive Approach to Assessing Student Learning in Mathematics

These test specifications focus on the development of the on-demand state level assessment in mathematics for grades 3 – 8 to meet the requirements of NCLB. The assessments, however, are a part of a larger system envisioned by the TSNE partners. Partner states envision a system that includes local summative and ongoing assessment (formative and summative) that are purposely embedded in the instructional program to provide teachers with the information that they need to take immediate action. However, the state-level on-demand assessment must support good instructional practices. These specifications were written from the perspective that large-scale assessment should promote and support good instructional and curriculum practices. “Large scale assessments should be substantially consistent with high quality classroom assessments though procedurally separate.” Shepard 2000.

The TSNE partners are at different stages in implementing this vision, but all believe in the importance of the state level assessment for supporting and promoting good instructional practices.

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Specification of the Tri-State New England (TSNE) Grade Level Expectations and TSNE Grade Level Assessments

The TSNE partners are committed to assessments that support good instructional practices, are explicitly specified to the TSNE GLEs, and provide access to the greatest number of students possible.

This specification started with the development of the TSNE GLEs that included the application of a set of criteria adapted from one of the partner states (Vermont) for development of TSNE GLEs. (See Appendix A.) Specification includes:

- The specific concepts and skills identified in the GLEs;
- The Distribution of Emphasis (DoE) identified across and within mathematics content clusters: Number and Operations; Geometry and Measurement; Functions and Algebra; and Data, Statistics, and Probability;
- The implied cognitive demand of the GLEs (Depth of Knowledge (Webb) and NAEP Levels of Complexity) and the “ceiling” identified;
- The interaction of the content with the cognitive demand; and
- *Conservation of Mathematical Construct* while allowing access to the greatest number of students (Lager and Petit, 2003) in Item Development.

To support the application of these specifications the content design teams conducted a number of studies to prioritize the concepts and skills articulated in the mathematics GLEs and provide contractors with the relevant data and background. Table 15 on pages 35-37 summarizes the rationale and Bidders flexibility in applying the protocols for each component of these test specifications. In addition, there is a section at the end of each set of components titled - Bidders Flexibility – with this information.

The TSNE Mathematics Test Specifications are organized into the following sections.

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A. Mathematics Content Strands and Reporting Areas

TSNE grade level assessments for grades 3 – 8 will be administered in the fall of grades 3 – 8 drawing on the GLEs from the previous grade. Table M.1 indicates the number of GLEs for each grade and content strand based upon the October 1, 2003 Field Review Version of the Draft Compact Mathematics GLEs. TSNE Mathematics GLEs are organized into four content strands: Number and Operations; Geometry and Measurement; Functions and Algebra; and Data, Statistics and Probability.

Table M.1: Number* of TSNE GLEs by Grade and Content Strands – February 12, 2004

Number of Mathematics GLEs by Grade						
	2	3	4	5	6	7
Number and Operation	4	4	4	4	4	3
Geometry and Measurement	3	3	5	5	5	5
Functions and Algebra	2	2	3	3	4	4
Data, Statistics, and Probability	3	4	4	4	4	4
Total by Grade	12	13	16	16	17	17

* The number of GLEs cannot be directly translated into the relative emphasis on the content strands at a given grade level. See the Distribution of Emphasis Table M.3 for relative emphasis and recommended distribution of items across content strands and within content strands.

Reporting areas:

The TSNE Mathematics Grade Level Assessments will yield an overall performance score in four performance levels to be established by the TSNE partners. It will also yield raw scores (number of items correct or percent of points earned) for each content strand that has 10 or more possible points.

Bidders' Flexibility in regard to the content strands:

- 1) Raw scores or percent of points earned shall be reported for any content strand that has 10 or more points.
- 2) Bidders should provide methods of reporting the raw data for the content strands and any other diagnostic information that can be derived from the assessment.

B. Format of Mathematics GLEs

(Note: The Criteria for the Development of TSNE GLEs is found in Appendix A. Potential bidders should review the TSNE GLEs in light of these criteria.)

The set of GLEs within and across a grade span and within a content cluster have specific features that developers need to address to assure alignment to TSNE mathematics GLEs. The features include: (1) the “stem”; (2) specifics related to the stem; (3) differences

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identified between adjacent grade levels; and, (4) the use of conjunctions “and,” “or,” “or^{sc}.”

B. 1: The Stem

Each TSNE Mathematics’ GLE contains a ***bolded*** statement – the stem. These statements identify the big ideas related to the content strand that the TSNE has identified for inclusion in the large-scale state assessments. These statements are designed to help “focus” the state assessment without narrowing the curriculum and instruction supporting student learning related to the GLEs. The set of “stems” within a content strand provide a guide to maintain coherence in both curriculum and assessment.

Table M.2 contains GLEs for grades 5 and 6 for the Number and Operations strand. There are four “stems” that span across grades 2 –8. These stems focus on the big ideas of Number and Operation as identified by the TSNE partner state mathematics design committee.

Table M. 2: Grades 5 and 6 TSNE Number and Operation GLEs

Grade 5	Grade 6
<p>M(N&O)–5–1 Demonstrates conceptual understanding of rational numbers with respect to: whole numbers from 0 to 9,999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations; and</p> <p>positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, <u>twelfths</u>, fifths, or powers of <u>ten</u> (10, 100, 1000)), decimals (to thousandths), or benchmark percents (10%, 25%, 50%, 75% or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.*</p>	<p>M(N&O)–6–1 Demonstrates conceptual understanding of rational numbers with respect to ratios (comparison of two whole numbers by division a/b, $a : b$, and $a \div b$, where $b \neq 0$); <u>equivalence across number formats</u>; and <u>rates</u> (e.g., a out of b, 25%) using models, explanations, or other representations*.</p>
<p>*Specifications for area, set, and linear models for grades 5 – 8: Fractions: The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator. Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). Decimals (including powers of ten): The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.</p>	
<p>M(N&O)–5–2 Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying equivalent positive <u>fractional numbers</u>, decimals, or <u>benchmark percents within number formats</u> (fractions to fractions, decimals to decimals, or percents to percents); or <u>integers</u> in context using models or number lines.</p>	<p>M(N&O)–6–2 Demonstrates understanding of the relative magnitude of numbers by ordering or comparing <u>numbers with whole number bases and whole number exponents</u> (e.g., 3^3, 4^3), integers, or <u>rational numbers within number formats</u>; and <u>comparing multiples of 10% and 25% up to and including 100% across number formats</u> using number lines or <u>equality and inequality symbols</u>.</p>
<p>M(N&O)–5–3 Demonstrates conceptual understanding of mathematical operations by describing or illustrating the <u>meaning of a remainder with respect to division of whole numbers</u> using models, explanations, or <u>solving problems</u>.</p>	<p>M(N&O)–6–3 Demonstrates conceptual understanding of mathematical operations by describing or illustrating the <u>meaning of a power by representing the relationship between the base (whole number) and the exponent (whole number)</u> (e.g., 3^3, 4^3); and the effect on the magnitude of a whole number when <u>multiplying or dividing it by a whole number, decimal, or fraction</u>.</p>
<p>M(N&O)–5–4 Accurately solves problems involving multiple operations on whole numbers or the use of the properties of factors, multiples, <u>prime, or composite numbers</u>; and addition or subtraction of <u>fractions (proper) and decimals to the hundredths place</u>. (Division of whole numbers by up to a two-digit divisor.)</p> <p>(IMPORTANT: Applies the conventions of order of operations <u>with and without parentheses</u>.)</p>	<p>M(N&O)–6–4 Accurately solves problems involving single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; <u>percent of a whole</u>; or problems involving <u>greatest common factor or least common multiple</u>.</p> <p>(IMPORTANT: Applies the conventions of order of operations <u>with and without parentheses</u>.)</p>

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B.2: Specifics and Differences Between Adjacent Grades

The unbolded portion of the GLE in conjunction with the stem identifies the specific content and implied cognitive demand at a given grade level. Differences between adjacent grades are underlined. In the example below students at the beginning of grade 7 are expected to extend a variety of patterns to specific cases like grade 5 students, but they are also expected to generalize and express that generalization using words or symbols.

Grade 5 (Beginning 6)	Grade 6 (Beginning 7)
M(F&A)–5–1 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, or <u>in problem situations</u> ; and writes a rule in words or ^{sc} symbols <u>for finding specific cases of a linear relationship</u> .	M(F&A)–6–1 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, <u>graphs</u> , or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or <u>writes a rule in words or^{sc} symbols for finding specific cases of a nonlinear relationship</u> ; and <u>writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a linear relationship (e.g., twice the term number plus 1 or^{sc} $2n + 1$)</u> .

B.3: Conjunctions – “and,” “or,” and “or^{sc}”

The use of the conjunctions in the TSNE mathematics GLEs are specified for assessment development with the following meaning. “And” means that *to the extent possible* elements within a GLE connected by “and” should be included in the assessment every year. “Or” means that those aspects of the GLEs can vary from year to year.

While students will have choices on strategies they use or methods to communicate their thinking throughout the assessment, there are special cases that the TSNE partners thought it was necessary to communicate to the test developer that students should not be required to use a specific method (e.g., “...writes in words or^{sc} symbols...”). In these cases “or^{sc}” appears in the GLEs.

In GLE M–F&A–6–1 students are to be given the choice as to the way they communicate the generalization... uses words or^{sc} symbols.

.M(F&A)–6–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or writes a rule in words or^{sc} symbols for finding specific cases of a nonlinear relationship; and writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a linear relationship (e.g., twice the term number plus 1 or^{sc} $2n + 1$).

Sometimes this notation is found in relation to using strategies to solve a problem.

M(DSP)–4–4 **Uses counting techniques to solve problems** in context involving combinations or simple permutations (e.g., Given a map – Determine the number of paths from point A to point B.) using a variety of strategies (e.g., organized lists, tables, tree diagrams, or^{sc} others).

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In this case test developers are NOT to require the use of any of these strategies provided in the list.

Sample Items to illustrate application of “and,” “or,” and “or^{sc}.”

Examples I.1 and I.2 feature three important aspects of TSNE GLE M(F&A)–6–1: Application of “and,” “or,” and “or^{sc}.”

M(F&A)–6–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or writes a rule in words or^{sc} symbols for finding specific cases of a nonlinear relationship; and writes an expression or^{sc} equation using words or^{sc} symbols to express the **generalization** of a linear relationship (e.g., twice the term number plus 1 or^{sc} $2n + 1$).

In the Grade 6 (Beginning grade 7) grade level assessments would include items that assess student ability to identify and extend patterns to specific cases and to generalize patterns *every year*. However, the way that the patterns are represented may vary across years. (“**a variety of patterns** represented in models, tables, graphs, sequences, or in problem situations.”). In addition, the student is provided the option to express the generalization with words or symbols.

Example I.1: Pattern represented in a table and “or^{sc}”

Write a rule using words or symbols for the n^{th} term in the following arithmetic sequence. Justify your rule.

Position in Sequence	1	2	3	4	N
Term	5	9	13	17	?

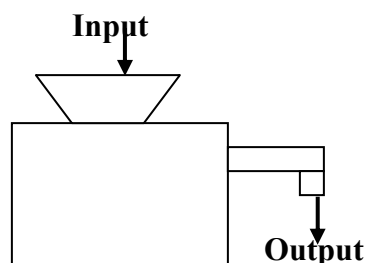
Use of model and “or^{sc}”

The input – output machine accompanied with a table of values is a familiar model used in elementary and middle school instruction. The students in this problem are required to identify the pattern in the data, extend to 3 specific cases, and provide a rule/generalization of the linear relationship. There is no requirement for how the student expresses the rule: words or^{sc} symbols.

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Example I.2: Item from NH Grade 6 2002 NHEIAP

33. Greg has been asked to find the rule for the function machine shown below. He inputs four numbers as a trial to see how the machine works. The results of the trial are shown in the table below.



Input	Output
3	13
7	29
2	9
10	41

- Using the same machine, if Greg inputs the number 5, what will the **output** be?
- Using the same machine, if the input is 12, what is the **output**?
- Using the same machine, what **input** would give an output of 61?
- If the input is the variable n , write the rule that shows the **output**.

B.4 Specific GLE Considerations:

The following are specifications for some GLEs to be used for item development and scoring guidelines. They are included in this section because the mathematics content team identified them as needing more clarity to assure that items and rubrics developed for the GLEs are consistent with the intent, or because they were identified in the national review by WestEd as needing more specification for item writers. The GLE specifications below are preliminary and should be refined by the mathematics content team working with the contractor in the development of the TSNE Assessment.

Number and Operations

GLE Specification 1: Items developed for the Number and Operation strand should include only those concepts and skills identified in the number strand. Other Number and Operations concepts developed across grades can be used in problems in the other content strands. (E.g., Although operations on decimal numbers is not identified at grade 7 for assessment in the Number and Operation strand, decimals can and should be used in problems associated with the other strands at grade 7.)

GLE Specification 2: The following statement applies to GLE stem M(N&O)–X–1 at grades 5 – 8. In general, this specification clarifies the models to use to represent the part-whole relationship for fractions, decimals, or percents, and is consistent with cognitive literature¹ on the conceptual development of these mathematical concepts. While the specification is included on the TSNE GLE document and for the most part self-explanatory, the mathematics content team felt that the specification for percents needed additional explanations.

¹ (Post et al, 19XX)

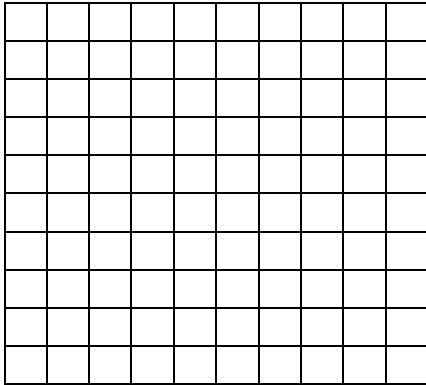
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Specifications for area, set, and linear models for grades 5 – 8: Fractions: The number of parts in the whole are equal to the denominator, a multiple of the denominator, or a factor of the denominator.

Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8). **Decimals (including powers of ten):** The number of parts in the whole is equal to the denominator of the fractional equivalent of the decimal, a multiple of the denominator of the fractional equivalent of the decimal, or a factor of the denominator of the fractional equivalent of the decimal.

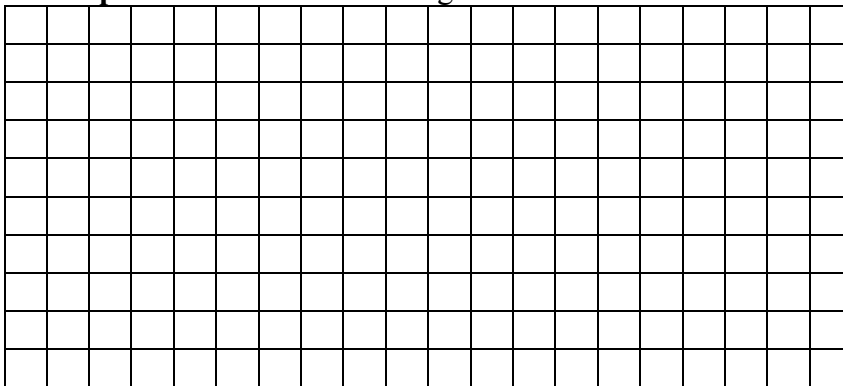
Percents: The number of parts in the whole is equal to 100, a multiple of 100, or a factor of 100 (for grade 5); the number of parts in the whole is a multiple or a factor of the numeric value representing the whole (for grades 6-8).

Example I.3: Shade 50% of the grid.



In this example the "...parts in the whole is equal to 100..."

Example I.4: Shade 50% of the grid.



In this example the "...parts in the whole is a multiple of 100..."

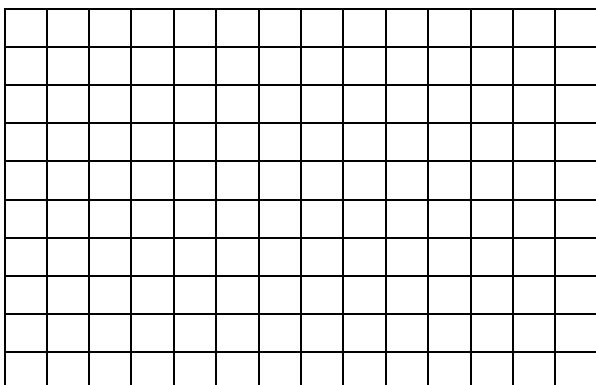
Example I.5: Shade 70% of the grid.



In this example the "...parts in the whole is a factor of 100..."

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Example I.6: Shade 50% of the grid.



In this example the "...the number of parts in the whole is a multiple or a factor of the numeric value representing the whole...." The numeric value of the whole is equal to 140 square units.

Note: Area models were used in these examples. However, test contractors should use all three models – area, set, and linear (e.g., number lines, rulers) at grades 5 – 8.

Functions and Algebra

GLE Specification 3: To the right find a more tightly specified TSNE GLE M(F&A)–7–1 than is currently found in the TSNE Mathematics GLEs for the purposes of designing items and scoring guides. These specifications consider the following three conditions as they relate to the representation of generalizations of linear and non-linear relationships (in words or with symbols) at different grade levels.

- 1) Expresses a generalization of a linear relationship using non-recursive explicit equations (Grade 7(fall 8));
- 2) Expresses a generalization of a non-linear relationship using an expression in open or^{sc} closed forms (grades 2 – 7(fall 8) and
- 3) Expresses a generalization of a non-linear relationship in an equation using a recursive equation or^{sc} non-recursive open or^{sc} closed explicit or^{sc} implicit equation (all grades except grade 8).

M(F&A)–7–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes a linear relationship (non-recursive explicit equation) using words and symbols; or generalizes a linear relationship to find a specific case; or writes an expression (open or^{sc} closed forms) or^{sc} equation (recursive equations or^{sc} non-recursive open or^{sc} closed explicit or^{sc} implicit equation) using words or^{sc} symbols to express the generalization of a nonlinear relationship**

GLE Specification 4: Prior to grade 7 expressions of generalizations for linear or non-linear relationships can be in the open or closed form - "writes an expression (open or^{sc} closed forms) or^{sc} equation (recursive equations or^{sc} non-recursive open or^{sc} closed explicit or^{sc} implicit equation) using words or^{sc} symbols to express the generalization."

Scoring Implications for GLE Specification 3 and 4.

- 1) At all grades prior to grade 7 students should receive all the points assigned to an item assessing the representation of a generalization (linear or non-linear) if they write an expression in open or closed form, or if they write recursive or non-recursive equations. (Assumes the representation is consistent with the pattern or relationship)
- 2) At grade 7 only non-recursive explicit equations representing generalizations of linear relationships should be given all the points assigned to an item assessing

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the representation of a generalization of a linear relationship. (Assumes the representation is consistent with the pattern or relationship)

- 3) At grade 7 students should receive all the points assigned to an item assessing the representation of a generalization (non-linear) if they write an expression in open or closed form, or if they write recursive or non-recursive equations. (Assumes the representation is consistent with the pattern or relationship)

Sample for Review of Scoring Representations of Generalizations

Note: The mathematics team did **NOT** make decisions about the number of points to assign different types of items. The two points is used to exemplify the current thinking of the mathematics content team in regards to scoring questions in which students are asked to represent linear and non-linear relationships.

	Grade 6 and below	Grade 7
Linear Relationship	2 – Expression or equation (word or ^{sc} symbols) accurately reflects the linear relationship, and the expression is in open or closed form, or the equation is recursive or non-recursive. 1 – 0 -	2 –Equation accurately reflects the linear relationship, and is a non-recursive explicit equation. 1 – Expression or equation accurately reflects the linear or non-linear relationship, and the expression is in open or closed form, or the equation is recursive or non-recursive. 0 -
Non-linear Relationship	2 – Expression or equation (word or ^{sc} symbols) accurately reflects the non-linear relationship, and the expression is in open or closed form, or the equation is recursive or non-recursive. 1 – 0 -	2 – Expression or equation (word or ^{sc} symbols) accurately reflects the non-linear relationship, and the expression is in open or closed form, or the equation is recursive or non-recursive. 1 – 0 -

GLE Specification 5 - Clarification recommended by WestEd: The Table below indicates the linear and non-linear patterns or relationships that are fair game at grades 2 – 7. (Note: Contractors should review these parameters with the New England partners to finalize them. They were developed as a result of comments from WestEd reviewers to clarify differences across adjacent grades. However, partner states have not had the opportunity to review them as of this writing.)

	2	3	4	5	6	7
Linear	$y = x + 1$	$y = x \pm b$ (b not greater than 3)	$y = x \pm b$ or $y = mx$	$y = x \pm b$ or $y = mx$ or $y = x/m$	$y = mx \pm b$ $y = x/m$	$y = mx \pm b$; $y = x/m \pm b$
Non- linear	Sequences involving addition	Sequences involving addition or subtraction	Sequences involving addition or subtraction	$y = x^2$	$y = x^2 \pm b$	$Y = x^n$ where n is a whole number 3 equal to or less than 3 and more than 1; $y = mx^2 \pm b$; $y = x^n \pm b$ where $n > 3$ or less than 1;
Non-numerical	AB, BBA etc	AB, BBA etc				

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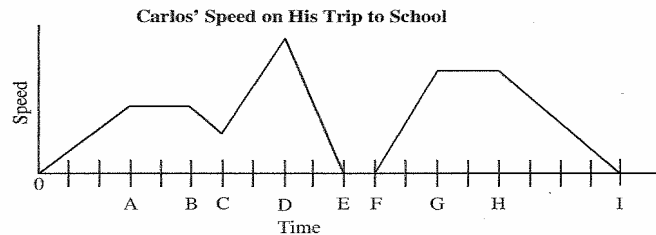
GLE Specification 6: To provide clarity to test contractors and to the field examples are found below to exemplify the intent of elements of GLE M(F&A)–7–2. The intent in this GLE is to assess developing understanding of slope and intercept in an informal way. It is not the intent to assess formal algebraic approaches for determining slope and intercept.

(Important Note: While some of these examples were drawn from a sample identified by the New England partners from released tasks, the content teams working with contractors should review these examples to reaffirm that they exemplify the intent of GLE M(F&A)–7–2. In addition, some items were written to fill gaps in aspects of the GLE not addressed by the sample drawn by the mathematics content team. These items are identified with an asterisk (*) and have not been reviewed by the mathematics content team or been piloted with students to assure they elicit the mathematics intended.)

M(F&A)–7–2: **Demonstrates conceptual understanding of linear relationships** ($y = kx$; $y = mx + b$) **as a constant rate of change** by solving problems involving the relationship between slope and rate of change, by describing the meaning of slope in concrete situations, or informally determining the slope of a line from a table or graph; **and distinguishes between constant and varying rates of change in concrete situations represented in tables or graphs**; or **describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.**

Example 1.7:

12. The graph below shows Carlos' speed on his trip to school.



“... by describing the meaning of slope in concrete situations...”

Based on the graph, when is Carlos' speed decreasing most rapidly?

- A. for times between B and C
- B. for times between D and E
- C. for times between E and F
- D. for times between H and I

Grade 8 MCAS 2002

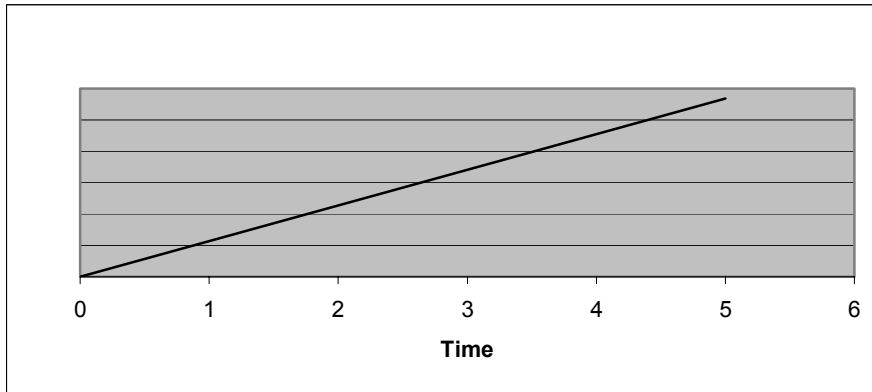
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Example 1.8: * Which of the following could be shown by the graph below?

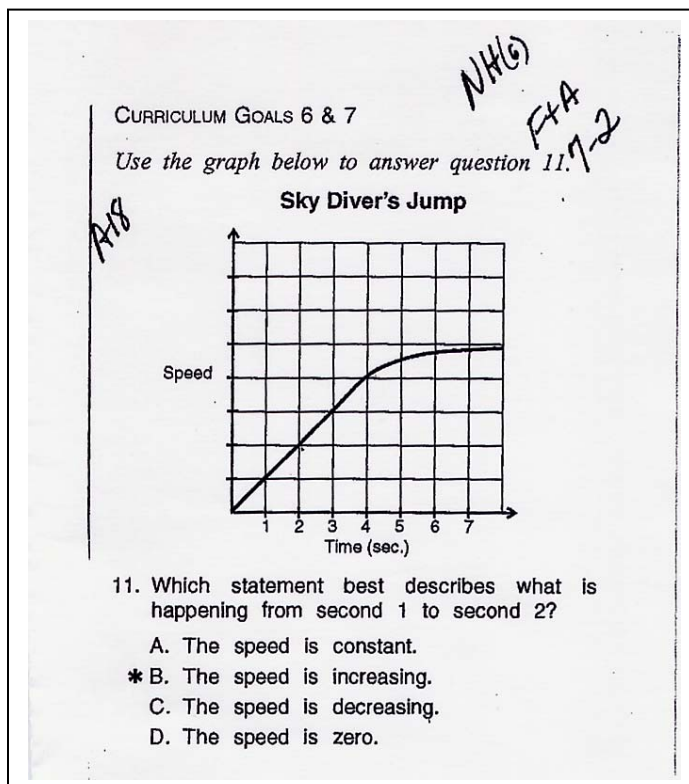
- a) The height of a candle as it burns.
- b) The height of a ball as it is thrown in the air.
- c) The distance covered by a car traveling at a constant rate.
- d) The height of water in a tank as it is being drained.

Explain your choice.

“... by describing the meaning of slope in concrete situations...”



Example 1.9:



“... by describing the meaning of slope in concrete situations...”

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
Example 1.10:


Mathematics, Grade 8


MCAS F+4 7-2


A12

4. Kendra is observing how the height of a tall, thin cylindrical candle changes over time as it burns. Which graph best represents the changes in the height of the candle over time?

A. 

B. 

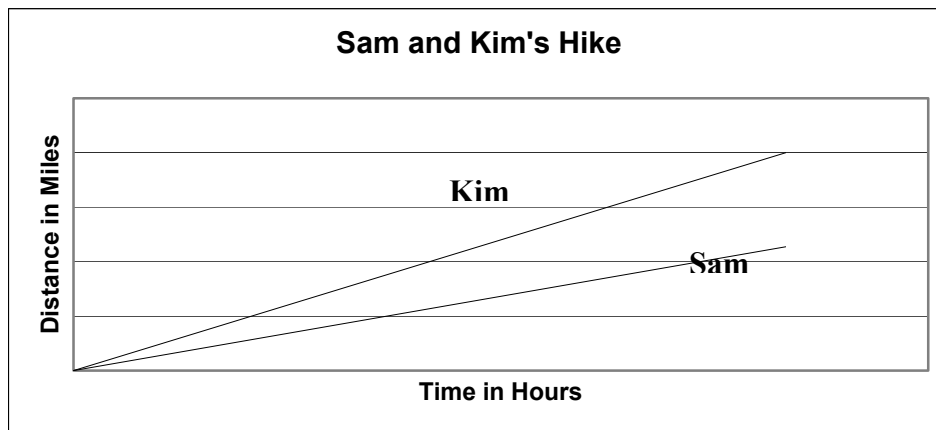
✓ C. 

D. 

Reporting Category for Question 4: Patterns and Functions (p. 135)

“... by describing the meaning of slope in concrete situations...” ... and distinguishes between constant and varying rates of change in concrete situations represented in tables or graphs.

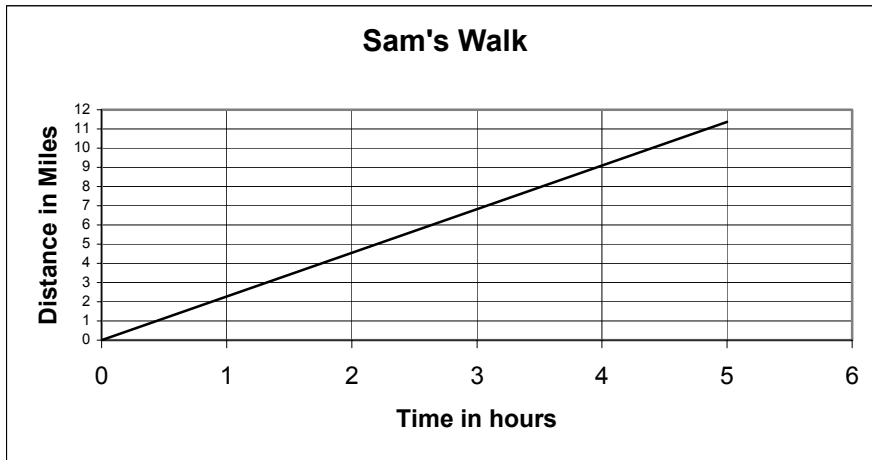
Example 1.11: * Who is hiking at the fastest speed – Kim or Sam? Explain how you know.



“... by describing the meaning of slope in concrete situations...”

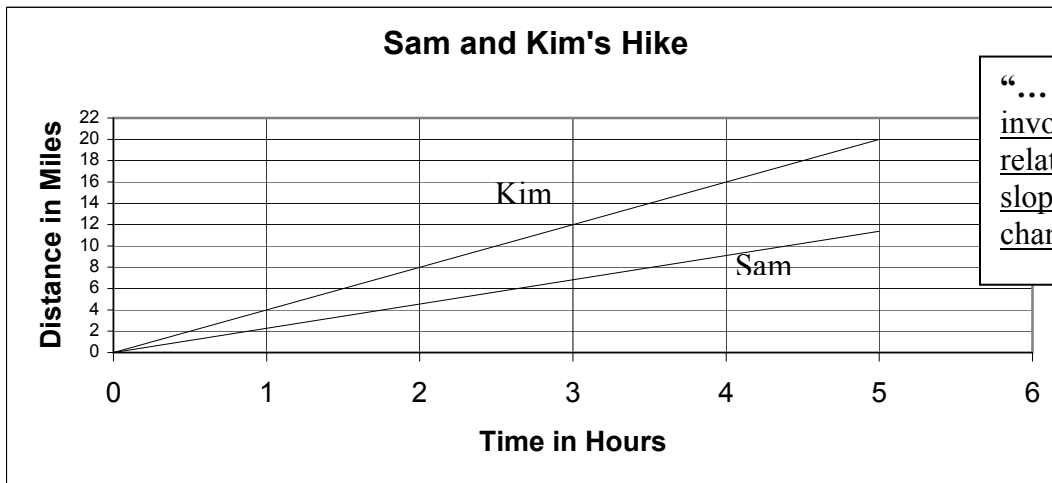
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Example 1.12: *At what rate is Sam walking?



“...or informally determining the slope of a line from a table or graph..”

Example 1.13: * Sam is going on a 30-mile hike. Kim is going on a 44-mile hike. If Sam and Kim continued to hike at the rates shown in the graph below, who will finish their hike first, and by how much time? Show your work.



“... by solving problems involving the relationship between slope and rate of change.”

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Example 1.14: The following equation shows the relationship between a distance traveled (d), the time traveled (t), and the rate (r):

$$\frac{d}{t} = r$$

If the time increases and the distance remains the same, what happens to the rate?

- a) It increases.
- b) It decreases.
- c) It remains the same.
- d) There is not enough information given.

(MCAS 2002 release item – grade 8)

“...describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.”

Geometry and Measurement

GLE Specification 7 - Clarification recommended by WestEd: - In GLE... “M(G&M)–2–1 Uses **properties, attributes, composition, or decomposition to sort or classify** polygons or objects by a combination of two or more non-measurable or measurable attributes.” ...the reference to “non-measurable attributes” includes the attributes of color, size, and number of lines and angles.(e.g., Identify all the small red rectangles.)

GLE Specification 8 - Clarification recommended by WestEd: In GLE... “M(G&M)–5–4 Demonstrates conceptual understanding of congruency by matching congruent figures using reflections, translations, or rotations (flips, slides, or turns), or as the result of composing or decomposing shapes using models or explanations.” ...transformations can include multiple transformations (flip and slide).

GLE Specification 9 - Clarification recommended by WestEd: In GLE... “M(DSP)–2–4 Uses **counting techniques to solve problems** involving combinations using a variety of strategies (e.g., student diagrams, organized lists, tables, tree diagrams, or^{sc} others); (e.g., How many ways can you make 50 cents using nickels, dimes, and quarters?)” ... there can be up to ten combinations. In addition, items should NOT be developed that require a student to interpret a specific type of representation (e.g., tree diagrams) for this GLE.

GLE Specification 9 - Clarification recommended by WestEd: In GLE... “M(DSP)–7–2 Analyzes patterns, trends, or distributions in data in a variety of contexts by solving problems using measures of central tendency (mean, median, or mode), dispersion (range or variation), or outliers to analyze situations to determine their effect on mean, median, or mode; and evaluates the sample from which the statistics were developed (bias).” ... the reference to “variation” does not include standard deviation.

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Bidders' flexibility in regard to the application of "and," "or," "or^{sc}" in sampling GLEs:

1. Every GLE should be assessed by at least 1 item every year.
2. To the extent possible elements of GLEs connected with "ands" should be assessed every year.
3. Elements of the GLE connected with "or" can be rotated across years.
4. Contractors should develop items that provide the opportunity for students to select their own strategies or methods of communication throughout the assessment. Particular attention should be paid to GLEs where elements are connected by "or^{sc}."

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C. Prioritization and Distribution of Emphasis Across and Within Content Strands

Key to establishing TSNE Mathematics GLEs was two prioritizing strategies: forced choices (Petit, 2003), and the identification of a recommended Distribution of Emphasis for assessment across and within the content strands.

The criterion, Balance of Representation (Webb, 1997), is applied for alignment purposes. Webb defines Balance of Representation as follows: “For the Balance of Representation criterion to be met, the degree of importance of different ideas given in the assessments and expectations should be the same.” To meet Balance of Representation the TSNE grade level assessment must reflect the emphasis identified in the Distribution of Emphasis study. The preliminary findings established on September 22 and 23, 2003 are found in Table M.3 and M.6.

The TSNE Distribution of Emphasis across and within content strands is influenced by the number of GLEs within a content strand, but is *not* driven by this factor. National literature about curriculum focus at given grades, the specifics within the GLEs, the recommendation to include aspects of GLEs every year to the extent possible, or sampling protocols suggested by the use of “and” and “or” between elements of the GLEs are the defining factors.

The TSNE mathematics development team used a strategy “forced choices” (Petit, 2003) throughout the development process to help prioritize the specifics identified within a content strand, across content strands, and to make decisions related to the use of the conjunctions “and” and “or.” This prioritization continued during the September 22nd “Distributions of Emphasis” study.

The mathematics team used the questions below to help prioritize the concepts and skills for state assessment purposes. As bidders review the GLEs in preparation for their response to the RFP, they should indicate any additional prioritization of GLEs based upon these guiding questions or other prioritization strategies.

Questions to guide prioritization (Petit, 2003):

- 1) Is the concept or skill part of a big idea in the discipline? (E.g., proportionality)
- 2) Is the success on the concept or skill in a given grade essential for success in mathematics in subsequent grades?
- 3) Should the concept or skill be assessed at an earlier grade because success at that earlier grade is important for success at the given grade?
- 4) Is the concept or skill assessed adequately at an earlier grade?
- 5) Should the concept or skill be assessed at a later grade for state assessment purposes?
- 6) Is the concept or skill subsumed in other GLEs at that grade level? (E.g., Is the skill of applying the conventions of order of operations subsumed in the use of formulas and when solving multistep linear equations at grade 8?)
- 7) Are concepts or skills important for success in other disciplines in given grades or subsequent grades?
- 8) Can the concept or skill be more appropriately assessed at the classroom level?

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Table M.3 contains the preliminary recommended Distribution of Emphasis on the assessment across content strands and grades. The percentages reflect estimates for contractor use in the development of the assessment. In general, more emphasis is placed on the Number and Operation strand at the earlier grades and less emphasis in middle school. On the other hand there is less emphasis on the Algebra and Functions strand in the earlier grades and more in middle schools.

Table M.3: TSNE Mathematics Preliminary Distribution of Emphasis (DoE) by Content Strand (September 22nd and 23rd, 2003.) *** The Distribution of Emphasis should be revisited to consider item distribution as result of changes in GLEs. These data are currently NOT consistent with the item distribution by GLEs found in Table M.6.

Note: 2(3) means grade 2 GLEs assessed in the beginning of grade 3

	Approximate Percent Distribution of Emphasis					
	2 (3)	3(4)	4(5)	5(6)	6(7)	7(8)
Number and Operations	55%	50%	50%	45%	30%	20%
Geometry and Measurement	15%	20%	20%	25%	25%	25 %
Algebra and Functions	15%	15%	15%	15%	30%	40%
Data, Statistics, and Probability	15%	15%	15%	15%	15%	15%
	100%	100%	100%	100%	100%	100%

The items available for assessment will be distributed across content strands and across GLEs within content strands consistent with the Distribution of Emphasis. Table M.4 is an example of how contractors might apply the Distribution of Emphasis to assignment of items to the four content strands to meet the Balance of Representation (Webb, 1997) criterion.

Table M.4: Example grade 2 and grade 7 - Preliminary Recommended Distribution of Items Across Content Strands Consistent with recommended Distribution of Emphasis if there are 50 items available (September, 2003)

	Approximate Number of Items			
	2 (Beginning grade 3)		7 (Beginning grade 8)	
	DoE	# of Items	DoE	# of Items
Number and Operations	55%	26	20%	10
Geometry and Measurement	15%	8	25 %	13
Algebra and Functions	15%	8	40%	19
Data, Statistics, and Probability	15%	8	15%	8
		50		50

As part of the strategy to further prioritize during the Distribution of Emphasis study, participants were asked to make preliminary decisions about the relative distribution of items across the GLEs within each content strand consistent with readings and the combinations of “and” and “ors” in a GLE as a result of prioritization.

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Table M.5 is a sample of the raw data generated from this exercise. The full set of raw data is available to potential bidders upon request. Table M.6 contains the relative balance in item distribution across the GLEs by grade and content strand. This should provide contractors information about the TSNE partners recommended sampling of the GLE. It assumes that some aspect of each GLE is assessed every year.

Table M.5: Sample Item Distribution (September 22, 2003)

Grades	Sample Item Distribution Data		
	2 (3)	3 (4)	4 (5)
M-N&O-X-1	7	5	4
	3	5	4
M-N&O-X-2	5	4	2
M-N&O-X-3	8	4	4
M-N&O-X-4		8	8
M-N&O-X-5	3		

Yellow – high emphasis;
gray – no GLE;
Purple – low emphasis;
;

Unshaded – equal emphasis between other unshaded, but less emphasis than yellow and more than purple

Table M.6: Recommended emphasis across GLEs within Content Strands

Yellow – high emphasis; Gray – no GLE; Purple – low emphasis; Unshaded – equal emphasis between other unshaded, but less emphasis than yellow and more than purple

****** These data have not been fully updated since the modification based on state field and national reviews, but give a general direction of the direction. ******

GLE #s	Preliminary Recommended Relative Emphasis of GLEs within Content Strands					
	2(3)	3(4)	4(5)	5(6)	6(7)	7(8)
M(DSP)-X-1	4	3	4	4	2	2
M(DSP)-X-2	2	1	1	1	2	2
M(DSP)-X-3		1		1		1
M(DSP)-X-4	2		1		1	
M(DSP)-X-5		1	1	2	2	3
M(F&A)-X-1	5	5	3	3	3	5
M(F&A)-X-2					7	7
M(F&A)-X-3			1	1	2	2
M(F&A)-X-4	3	3	2	3	4	5
M(G&M)-X-1	4	4	3	3	2	2
M(G&M)-X-2						2
M(G&M)-X-3			1	2	2	
M(G&M)-X-4				2		1
M(G&M)-X-5			1		3	4
M(G&M)-X-6	2	3	4	4	4	4
M(G&M)-X-7	This GLE will NOT be directly assessed but embedded in problems in other content strands. See the note below.*					
M(N&O)-X-1	7	5	4	8	5	4
	3	5	6			
M(N&O)-X-2	5	4	2	3	2	1
M(N&O)-X-3	8	4	4	3	4	
M(N&O)-X-4		8	9	8	4	5
M(N&O)-X-5	3					
Totals	48 + 2	47 + 3	47 + 3	48 +	49 + 1	50

*Note: Because of the shift in the assessment of M-G&M-X-7, there are additional items available to be used to assess other GLEs. Grade 2 - 2 items; Grade 3 – 3 items; Grade 4 – 3 items; Grade 5 – 3 items; and grade 6 – 1 item.

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Bidders' flexibility in regard to Sampling, Prioritization, Distribution of Emphasis:

- 1) Some aspect of each GLE should be assessed every year.
- 2) The distribution of items across the content strands should be consistent (to the degree possible +/- 5%) with the recommended TSNE Distribution of Emphasis.
- 3) The TSNE would not expect that the number of items across GLEs be rigidly adhered to from year to year, but does expect that the relative emphasis be attended to in sampling. Because the TSNE is interested in some GLEs being assessed deeper with more items, TSNE would **not** accept a GLE designated as low emphasis receiving high emphasis.
- 4) The relative balance as reflected by the number of points possible will vary from the Distribution of Emphasis across and within content strands as items types with their different point values are distributed across the GLEs. Contractors should provide strategies that consider the maintenance of Distribution of Emphasis for both items and points.

Contractors should indicate strategies to assure that the TSNE meets the Balance of Representation (Webb, 1997) criterion.

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D. Implied Cognitive Demand and Depth of Knowledge (Webb, 2002) and NAEP Levels of Complexity (NAEP 2004)

In order to define descriptors for cognitive demand to guide item development and classification of items, and link to the TSNE GLEs the TSNE partners will draw upon the work of Webb (1997, 2003), NAEP (2004) level of Complexities, and the implied cognitive demand in TSNE GLEs. These levels will be used to identify “ceiling” to guide item and overall test development, and potential levels for assessment.

An important aspect of the TSNE grade level assessment design is to use the highest Depth of Knowledge/Levels of Complexity demand implicit in a GLE as the “ceiling” for assessment, not the “target.” The “ceiling” defines the highest levels of assessment of a GLE and the other levels with potential for assessment. The “target” assumes that only the highest level is assessed.

Table M.7: Ceiling

Grade 6 (Beginning 7)	Ceiling	Potential Levels for Assessment to Ceiling
M(F&A)–6–1 Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, <u>graphs</u> , or in problem situations; or writes a rule in words or symbols for finding specific cases of a linear relationship; or <u>writes a rule in words or^{sc} symbols for finding specific cases of a nonlinear relationship</u> ; and <u>writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a linear relationship (e.g., twice the term number plus 1 or^{sc} $2n + 1$).</u>	3	2 – extends a pattern to a specific case 3 – generalizes a pattern

Why is the distinction between “ceiling” and “target” important? If one assessed only at the “target,” all GLEs with a level 3 as their highest demand would only be assessed at level three. This would potentially have two negative impacts on the assessment: 1) The assessment as a whole would be too difficult; and 2) important information about student learning along the achievement continuum would be lost (*Vermont Revised Draft Mathematics Test Specification, June 2003*).

In order to avoid these potential negative effects, the TSNE partners are specifying the ceilings for each GLE and distribution of Depth of Knowledge/Levels of Complexity across the assessment. The general protocol for this aspect is that a GLE should **not** be assessed above its “ceiling”, and to the extent possible at the “ceiling” and at least one level below the “ceiling.”

In April 2003, Norm Webb (email April 4, 2003) indicated that the current distribution of Depth of Knowledge used when applying the criterion in post hoc alignment analysis is at 50% of the items at a level 2 and above. However, he did not recommend a straight application of this distribution, but recommended that each state analyze their standards

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and related GLEs, and their vision to determine this distribution (*Vermont Revised Mathematics Test Specification, 2003*).

The TSNE partners have started this process.

The TSNE mathematics design team will review and build upon the Vermont Depth of Knowledge descriptors in *Vermont's Revised Draft Test Specifications* (June 2, 2003) (See Table M.8.).

Below find a general definition for each level. (Webb, 1999) Table M.8 contains preliminary descriptors based upon *Vermont Draft - Combined Webb Depth of Knowledge Levels* (Webb, 2002), *NAEP 2002 Mathematics Levels of Complexity*, and *Other Descriptors Related to Vermont GLEs*. These will be reviewed and modified in October by the TSNE mathematics team.

Level 1 involves recall, or the use of a procedure, or applying an algorithm or formula. It also includes one-step word problems, and other specifications unique to content standards.

Level 2 involves more than one step, demonstrating conceptual understanding through models and explanations, classifying information, and interpreting data from a simple graph.

Level 3 involves reasoning, planning and using evidence. Students would be asked at Level 3 to make and test conjectures, interpreting information from a complex graph, solve complex problems, explain concepts, and provide mathematical justifications.

Level 4 requires complex reasoning, planning, and thinking over extended periods of time. In mathematics, Level 4 Depth of Knowledge will not be assessed on the state grade level assessments, but will be assessed locally.

Table M.8: Vermont Draft - Combined Webb Depth of Knowledge Levels (Webb, 2002), NAEP 2002 Mathematics Levels of Complexity, and Other Descriptors Related to Vermont GLEs. (Subject to review by TSNE mathematics design team.)

Level 1	Level 2	Level 3
<ul style="list-style-type: none"> Recall or recognize a fact, definitions, or term Apply a well known algorithm Apply a formula Determine the area or perimeter of rectangles or triangles given a drawing and labels Identify a plane or three dimensional figure Measure a length Perform a specified procedure Evaluate an expression Solve a one-step word problem Retrieve information from a table or graph Recall, identify, or make conversations between and among representations or numbers (fractions, decimals, and percents), or within and between customary and metric measures Locate numbers on a number line, or points on a coordinate grid Solves linear equations 	<ul style="list-style-type: none"> Classify plane and three dimensional figures Interpret information from a simple graph Use models to represent mathematical concepts. Solve a problem requiring multiple steps, or the application of multiple concepts Compare figures or statements Compare and contrast figures Provide justifications for steps in a solution process Extend a pattern Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps Translate between tables, graphs, words and symbolic notation Direct translation between problem situations and symbolic notation. 	<ul style="list-style-type: none"> Interpret information from a complex graph Explain thinking Make and/or justify conjectures Develop logical arguments for a concept Use concepts to solve problems Perform procedure with multiple steps and multiple decision points Generalize a pattern Describe, compare, and contrast solution methods Formulate a mathematical model for a complex situation Provide mathematical justifications Solve a problem requiring multiple steps-supported with a mathematical explanation that justifies the answer. Translation between a problem situation and symbolic notation that is not a direct translation.

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Table M.9 contains the Preliminary “ceilings” and potential Depth of Knowledge Levels for assessment for each GLE.

Table M.9: Preliminary “Ceilings” and Potential Levels for Assessment

	Preliminary “Ceilings” and Potential Levels for Assessment					
GLE #s	2	3	4	5	6	7
M(DSP) -X-1	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M(DSP) -X-2	2,3*	2,3*	2,3*	2,3	2,3	2,3
M(DSP) -X-3		1,2		1,2		2,3
M(DSP) -X-4	2		2,3		2,3	
M(DSP) -X-5		1,2	1,2	1,2	1,2,3	1,2,3
M(F&A) -X-1	2	2	2	2	2,3	2,3
M(F&A) -X-2					1,2	1,2,3
M(F&A) -X-3			1	1	1,2	1,2
M(F&A) -X-4	1	1,2	1,2	1,2	1,2	1,2
M(G&M) -X-1	1,2,3	1,2	1,2	1,2	1,2	
M(G&M) -X-2						1,2
M(G&M) -X-3			1,2	1,2	1,2	
M(G&M) -X-4				1,2		1,2
M(G&M) -X-5			1,2		1,2	1,2,3
M(G&M) -X-6	1,2	1,2	1,2	1,2	1,2,3	1,2,3
M(G&M) -X-7	This GLE will NOT be directly assessed but embedded in problems in other content strands					
M(N&O) -X-1	1,2	1,2	1,2	1,2	1,2	1,2
	1,2	1,2	1,2			
M(N&O) -X-2	1	2	2	2	2	2
M(N&O) -X-3	1,2	2	2	2,3	2,3	
M(N&O) -X-4		1,2,3	1,2,3	1,2,3	1,2,3	1,2,3
M(N&O) -X-5	1,2					
Totals						

Bidders’ flexibility in regard to Depth of Knowledge:

- 1) Distribution of Emphasis of Depth of Knowledge across the assessment should stay within the general guidelines that the TSNE partners establish.
- 2) The general alignment protocol for this aspect is that a GLE should **not** be assessed above its “ceiling”, and to the extent possible at the “ceiling” and at least one level below the “ceiling.”
- 3) Bidders should include strategies to address the distributions across the levels.
- 4) Bidders should indicate any concerns or limitations that may arise from specification in # 2 above. Bidders can provide alternative strategies to address this specification that assures that the interaction of content with cognitive demand is incorporated into the design.
- 5) Bidders should be prepared to work with the TSNE content teams to solidify the TSNE Depth of Knowledge descriptors.

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E. Interaction of DoK and Content

In development, contractors should carefully consider the interaction of the content with the Depth of Knowledge. The graphic below considers this interaction. The x-axis represents the increasing complexity of the content. The y-axis indicates the Depth of Knowledge (DoK) level. The shaded box indicates the interaction. For example, graphic A represents content of low complexity interacting with the Level 1 Depth of Knowledge (*Vermont Revised Draft Test Specifications, June 2003*).



The TSNE does not expect its blueprint to require that all cells be filled, but it is intent on avoiding scenarios, except where otherwise specified, where items for single GLEs are concentrated in single cells, especially when the more advanced topics are assessed *only* at the highest Depth of Knowledge levels (Graphic B).

Consider the following case for GLE M–N&O–7–4 **Accurately solves problems involving proportional reasoning; percents involving discounts, tax, or tips; and rates.**

It is the intent that some of the problems sampling this GLE would be single step Level 1 problems, some multiple steps Level 2 problems, and a few Level 3 problems. In no case should all the problems sampling this GLE be at level 3, and definitely not all at just level 1.

Bidders' flexibility in regard to the Interaction of Content and Depth of Knowledge:

This element is a conceptual model. The TSNE does not expect contractors to fill all the cells in the models above, but does expect contractors to avoid scenarios where the hardest content is assessed at the highest level and easiest content at the lowest level.

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F. Item Types

There will be 4 item types in the TSNE Mathematics Assessment.

Table 10: Mathematics Item Types

	Estimated Time per Item	Description
Selected Response - MC (1 point each)	1 Minute	All multiple-choice items will consist of a stem and four response alternatives (i.e., a single correct response and three distractors). There will be a single correct response to each multiple-choice item. Response alternatives such as 'All of the above', 'None of the Above', 'A and B only' will not be used on the TSNE assessments.
Short Answer (1 points)	1 minute	Short-answer items will be constructed to provide students with an opportunity to demonstrate a skill that cannot be measured easily through the use of selected-response items. There will be a single correct response (or response set) to each short-answer item. However, there may be multiple methods for expressing the correct response. All short-answer items will include a scoring guide that includes all expected representations of the correct response.
Short Constructed Response (2 points)	2 minutes each	Constructed-response items will be developed to provide students opportunities to explain or show their reasoning and problem solving processes. Each constructed response item will include an item-specific analytic scoring guide that includes the following: detailed descriptions of required performance at each score point, annotated sample responses to demonstrate solid performance at each score point, annotated sample responses to demonstrate minimal performance at each score point.
Medium/Long Constructed Response (3 or 4 points)	8 Minutes	

Examples in this section are provided to illustrate the type of item and alignment to TSNE GLEs. They are drawn from released items from NHEIAP, MCAS, MEA, Texas, Missouri, *Vermont's Draft Revised Test Specifications* (June 2003), and written for these specifications. They have not undergone potential changes based upon accessibility considerations. On October 28th and 29th the TSNE mathematics team will meet to complete this work. These are preliminary examples.

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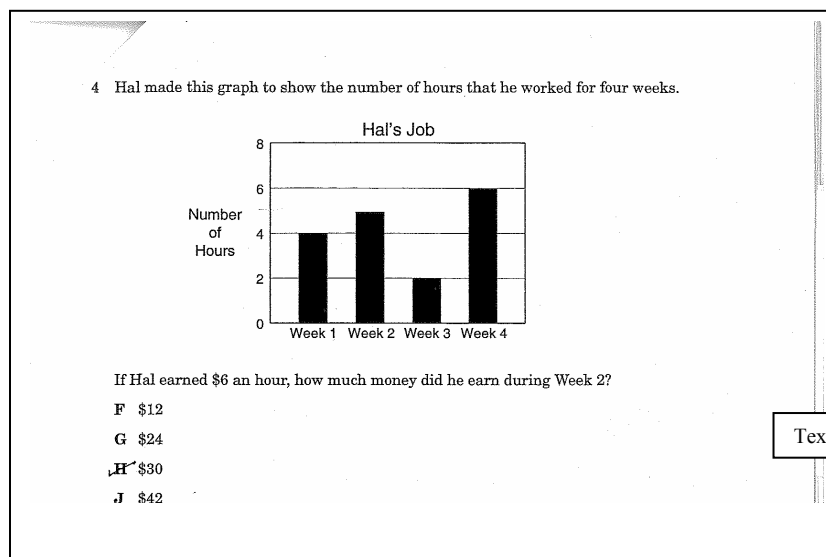
Table 11 provides the proposed distribution of item types in the common item portion of the assessment.

Table 11: Proposed Distribution of Item Types

	Estimated Time per Item	Number of Common Items
Selected Response - MC (1 point each)	1 Minute	34
Short Answer (1 points)	1 minute	6
Short Constructed Response (2 points)	2 minutes each	6
Medium Constructed Response (3,4)	8 Minutes	4
	Totals	50

Example I.15: Selected Response Item


M(DSP)–5–1 **Interprets a given representation** (tables, bar graphs, circle graphs, or line graphs) ...to solve problems.



Example I.16: Short Answer (1 point)

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M(F&A)–6–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions by... solving multi-step linear equations of the form $ax \pm b = c$, where a , b , and c are whole numbers with $a \neq 0$.

 What value of p makes the equation below true?
$$3p + 1 = 13$$

$3p$ means 3 times p

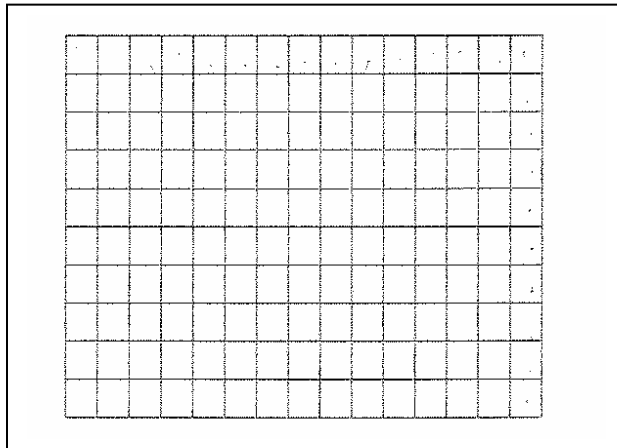
Grade 6 MCAS

Example I.17: Short Constructed Response (2 points)

(N&O)–6–1 **Demonstrates conceptual understanding of rational numbers with respect to ratios** (comparison of two whole numbers by division a/b , $a : b$, and $a \div b$, where $b \neq 0$)...**using models, explanations, or other representations.**

Shade 60 % of the grid.

Provide an explanation that supports why you shaded that portion of the grid.



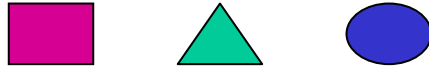
Vermont Revised Draft Test
Specifications, June 2003

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Example I.18: Medium/Long Constructed Response

M(F&A)–6–4 **Demonstrates conceptual understanding of equality** by showing equivalence between two expressions using models or different representations of the expressions...

38. Each shape below stands for a number. Each shape stands for the same number no matter where the shape is placed.



MEA Grade 4

a. Figure out what number each of the three shapes stands for in the number sentences below. Explain how you figured out the value for each shape.

$$\text{blue oval} + \text{blue oval} + \text{blue oval} + \text{blue oval} = 36$$

$$\text{green triangle} + \text{pink square} + \text{blue oval} = 13$$

$$\text{pink square} + \text{pink square} + \text{green triangle} = 5$$

b. Use as few shapes as possible to represent the number 25.

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Bidders' flexibility in regard to Distribution of the Item Types in the Common Items:

- 1) The assessment at each grade level must include a distribution of all four-item types. (See grade 3 exception in # 6 below.)
- 2) The total number of short answer and short constructed response items should be about 12; not necessarily 6 of each type.
- 3) The assessment at each grade level should include approximately 50 items to maintain adequate sampling. (See grade 3 exception in # 6 below.)
- 4) The assessment at each grade level should include 4 medium-long constructed response items. The current thinking is one for each content strand. (See grade 3 exception in # 6 below.)
- 5) The testing period cannot exceed 2 – one-hour sessions.
- 6) Contractors can propose alternative distributions of items for the beginning grade 3 assessment that might have fewer, or no, medium to long constructed response items. More short answer or constructed response may be used in their place.
- 7) Potential bidders should provide the TSNE partners with examples of rubrics to assess each of these item types. Each constructed response item will include an item-specific analytic scoring guide that includes the following: detailed descriptions of required performance at each score point.

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G. Conserving the Mathematical Construct (Petit and Lager, 2003) in Item Development while providing access to the greatest number of students

This component of the TSNE Test Specifications specifically addresses issues that relate to accessibility and the development of mathematics items that “conserve the mathematical construct” while providing access to the greatest number of students.

Similar to the Conservation of Mass principle in physics, the Conservation of the Mathematical Construct theory in assessment (Petit and Lager, 2003) provides guidelines for item writers to fine-tune the language, context, and format of an item to maximize its accessibility, without altering the mathematical construct being assessed.

Conserving the mathematical construct as defined by Petit and Lager (2003)² is the process of constructing assessment items that preserve the mathematical construct (content and cognitive demand) being assessed, while embedding the mathematics in rich contexts (when appropriate) and streamlining the language. The purpose of attending to both context and language simultaneously is to provide the greatest number of students the opportunity to demonstrate their knowledge and skills in relationship to the mathematical construct being assessed.

Conserving the mathematical construct road guidelines:

- 1) Explicitly aligning items with the content and cognitive demands in the TSNE GLEs;
- 2) Embedding the item in a rich context (when appropriate);
- 3) Streamlining the language; and
- 4) Appropriate use of graphics, pictures, graphs, tables, diagrams, and models.

A series of examples of simultaneously attending to linguistic and contextual issues while *conserving the mathematical construct* when writing mathematics assessment items is presented below. Although the examples serve to address the issues and provide examples of current thinking, the methods have not yet been thoroughly tested.

Explicitly aligning items to the mathematical construct

Example I.19 provides an example where the assessment item may or may not assess the mathematical construct being assessed in TSNE GLE M–F&A–7–1. Example I.7 is a modification of I.6 that brings it into alignment. Icons accompany these items to illustrate alignment or misalignment.

Aligned to mathematical construct in TSNE GLEs -



Not aligned to mathematical construct in TSNE GLEs -



² Unpublished work in progress by Marge Petit and Carl Lager. Please do not distribute or cite separately from this paper without permission.

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Item I.20 is not explicitly aligned with TSNE GLE as students can solve this problem by using substitution, not necessarily by deriving the generalization as required in the TSNE GLE.

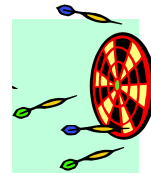
M(F&A)–7–1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression or^{sc} equation using words or^{sc} symbols to express the generalization of a nonlinear relationship.**

I.20:

- 29 Which expression can be used to find the n th term in the following arithmetic sequence, where n represents a number's position in the sequence?

Position in Sequence	1	2	3	4	n
Term	5	9	13	17	?

Modified – grade
8 – MCAS



- A $n + 4$
- B $3n + 4$
- C $5n$
- D $4n + 1$

Texas 2002 8th
Grade grade

Item I.21 is a modification of item I.20 that is explicitly aligned to TSNE GLE M–F&A–7–1 because the item requires that the student to make the generalization, not select from a list of potential generalizations.

Item I.21: Write a rule using words and symbols for the n^{th} term in the following arithmetic sequence. Justify your rule.

Position in Sequence	1	2	3	4	N
Term	5	9	13	17	?



Example I.22 is a multiple-choice question that explicitly assesses the mathematical construct of the element of TSNE GLE M–F&A–7–2 by selecting the segment of the graph that best describes Carlo's speed decreasing most rapidly (describes the meaning of slope in a concrete situation).

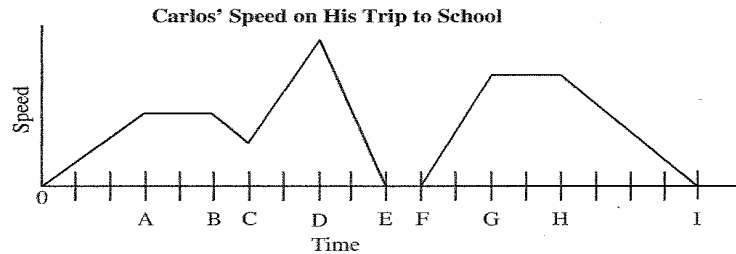
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M(F&A)–7–2 **Demonstrates conceptual understanding of linear relationships** ($y = kx$; $y = mx + b$) **as a constant rate of change ... by describing the meaning of slope in concrete situations ...**

Example I.22

12

The graph below shows Carlos' speed on his trip to school.



Based on the graph, when is Carlos' speed decreasing most rapidly?

- A. for times between B and C
- B. for times between D and E
- C. for times between E and F
- D. for times between H and I

Grade 8 MCAS 2002

Embedding the item in a rich context (when appropriate)

Mathematics instruction and assessment over the last decade has focused on embedding mathematics into meaningful contexts. There are many reasons why this strategy is important. DeLange (1987) indicates four important reasons.

Rationale for inclusion of applied context: (Jan deLange)

- 1) Allows students a natural and motivating access to mathematics;
- 2) Gives them a firm hold for learning formal operations and procedures;
- 3) Serves as a support for thinking; and
- 4) Shows a reality as a source and domain of application.

There have been many good examples in curriculum, instruction, and assessment of accomplishing these goals. There are also examples of inappropriate use or misuse of contexts. The TSNE partners wishes to avoid the following.

- 1) Over use;
- 2) Inappropriate inclusion for engagement, but not necessary to assess the construct;
- 3) Inappropriate context for the mathematics;
- 4) Ambiguous contexts; (Trying to get at one mathematical construct, but others can be equally inferred.); and
- 5) Context with cultural bias.

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Simultaneously attending to linguistics and contextual issues while conserving the mathematical construct

Examples I.23 – I. 28 provide examples and counter examples that address all four aspects of conserving the mathematical construct.

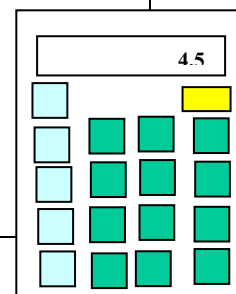
A mathematics task is presented below, first in original form³, and then in revised form reflecting essential linguistic and contextual considerations. (The example is a part of the unpublished work in progress by Marge Petit and Carl Lager. Please do not distribute or cite separately from these test specifications without permission.)

Example I.23: Sample mathematics task, original

Amy, Eric, and Kayla decide to share a pie that costs \$7.50. The 3 friends will split the cost of the pie equally. Eric used his calculator to find how much each of them should pay. The calculator display shows Eric's answer.

Eric got \$4.50 for an answer. Is \$4.50 a reasonable answer?

In the box below, explain why you think Eric's answer is or is not reasonable.



The first step in this process was to identify the mathematical constructs that were central to this assessment item, based on analysis of the item.

- Application of additive or multiplicative reasoning related to “equal” parts in a problem situation; and
- Application of additive and multiplicative reasoning using money.

Note: At first glance one might think that the item was designed to assess division (multiplicative reasoning). However, students could appropriately justify their choice using additive strategies.

There are two contexts that were identified in the item, one that is appropriate given the mathematics being assessed, and the other that is extraneous and unimportant.

³ Missouri Grade 4 mathematics released task, 1999. Note: the calculator graphic has not been reproduced accurately in this paper.

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Table 12: Contexts in item

Contexts	Action
The pie as a model for demonstrating understanding of division	Retain context, as it is a common model used in instruction for dealing parts/whole relationships.
The calculator (Unnecessary inclusion, but not necessary to assess the mathematical construct being assessed.)	The calculator was removed because it is irrelevant to a student demonstrating an understanding of the mathematics in this problem. Since \$4.50 is given, there is no reason for the calculator or the language associated with the calculator.

Next several design concerns with this item centered on linguistic concerns were identified that were potentially confusing or extraneous. Specific ways to address the design concerns are identified. (See Table 12.) Note that the first, third, fourth, and fifth concerns are primarily linguistic, and reflect language usage that a native English speaker might find transparent, but which an English language learner might find confusing. The second concern stems from extraneous information.

Table 13: Concerns and modifications for sample mathematics task

Design Concern	Modification
Referring to the persons in three ways – “Amy, Eric, and Kayla”, “3 friends,” and “them/each of them”	Eliminate the proper names as they are irrelevant both to the mathematics and the context of the task
Since \$4.50 is given, there is no reason for the calculator or the language associated with the calculator.	Eliminate the calculator graphic and all references to the calculator itself.
Referring to the claim in multiple ways – “Eric’s answer,” “\$4.50 for an answer,” “\$4.50 a reasonable answer”	Use quotation marks to highlight the claim to which the student is to respond
“Decide to share” – First, the task is not about sharing the pie, but splitting its cost equally. Second, the phrase “decide to...” is superfluous to the stated action (sharing).	Eliminate unnecessarily misleading, confusing and extraneous language.
Multiple verb tenses and forms are used to convey action – “share a pie” (present), “will split the cost” (future), “Eric used” (past), “should pay” (present conditional), “display shows” (present), “Eric got” (past), “Is \$4.50 a reasonable answer?” (present interrogative)	Within each task, use as few verb tenses and forms as possible. Present tense is usually the easiest for a student to cognitively process.

The revised task, reflecting the modifications for linguistic clarity and simplicity, is shown below (See Table 13.). Note that in addition to eliminating areas of concern, Petit

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and Lager reduced the number of words in the item without compromising the engaging problem context or the mathematics content.

Example I.24: Sample mathematics task, revision #1



Three friends buy a pie for \$7.50. They split the cost of the pie equally.

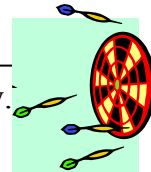
Your classmate says, “Each friend should pay \$4.50.”

Should each friend pay \$4.50? Explain why or why not.

Note: “Classmate” was used to purposely to make the claim by someone outside the original group of 3 friends. This allows one to eliminate double references like – “One of the friends says, “Each of us...”

The modification in this next example changed the mathematics being assessed even though it dealt appropriately with the linguistic issues. In the example students are being assessed on recognition of the situation being division, and accurately dividing.

Example I.25: Sample mathematics task, revision #2

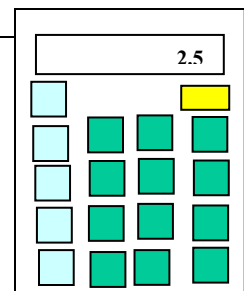


Three friends buy a pie for \$7.50. They split the cost of the pie equally.

How much money should each friend pay? Show your work.

Petit and Lager noted that while the calculator was not important and could be eliminated in the original version of the task, the task could be modified so that the calculator was relevant (Revision #3). Note that in addition to additive or multiplicative reasoning being assessed, there is an additional mathematical demand required. Students need to recognize the appropriate unit of measure (dollars) and/or convert the calculated quantity from decimals (2.5) to dollars (\$2.50).

Example I.26: Sample mathematics task, revision #3



Three friends buy a pie for \$7.50.

They split the cost of the pie equally.

Using a calculator, your classmate calculates that each friend pays 2.5.

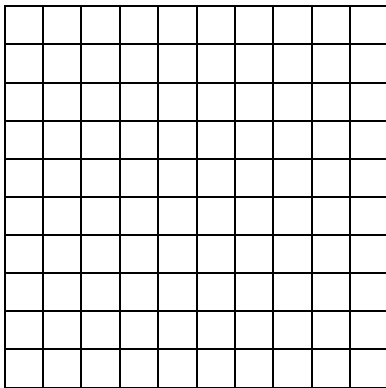
Is your classmate correct? Explain why or why not.

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The next series of examples illustrates how important it is to consider the mathematical construct when streamlining language. In example I.27 the student is required to shade 25% of the region on the grid. “Shade” cues the student to the action, 25% represents the part to be shaded, and “of the grid” defines the whole. Any further streamlining of the language – like just “Shade 25%” - would not conserve the mathematical construct because the whole would not be explicitly defined. In addition, the task is free of context. It is not necessary to assess this mathematics and would add extraneous language. The grid is a common instructional model.

Example I.27: Vermont’s Revised Draft Test Specifications, June 2, 2003

M(N&O)–5–1 **Demonstrates conceptual understanding of rational numbers with respect to ... benchmark percents (10%, 25%, 50%, 75%, or 100%) as a part to whole relationship... in area, set, or linear models **using models, explanations, or other representations.****



Shade 25% of the grid.



Explain why you shaded that part of the grid.

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Example I.28 is embedded in a context intentionally to provide students a “vehicle upon which to provide an explanation” (DeLange, 1987) of the mathematics in a meaningful context. The context is one that all students have a basis for understanding and experience (students who walk to school).

Example I.28:

M(N&O)–7–1 Demonstrates conceptual understanding of rational numbers with respect to percents as a means of comparing the same or different parts of the whole when the wholes vary in magnitude (e.g., 8 girls in a classroom of 16 students compared to 8 girls in a classroom of 20 students, or 20% of 400 compared to 50% of 100);...using models, explanations, or other representations

I.28: A headline in a local newspaper read:

Study shows more students walk to school in Westport than walk to school in Danville

Here is the data provided in the article.

	Percent of Students who Walk to School
Westport	70%
Danville	50%



Explain how it *could* be possible that a greater number of students walk to school in Danville, even though a greater percentage of students walk to school in Westport.

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Streamlining Language while conserving the mathematical construct

Some work has already been conducted to improve the language of mathematics items. What follows is a sample of previous work done in the field:

The TSNE partners expects to draw upon the work of Kopriva (2000) on readability and amount of text students are expected to use, Plain Language (Hanson, et al. 1998, Kopriva, 2000), and Abedi et al (2001a, 2001b) in regard to Simplified Language.

Abedi et al., (2001a, 2001b), for example, has a set of guidelines for simplifying language.

- a) Change unfamiliar or infrequent words to more familiar words
- b) Passive verb forms changed to active verb forms
- c) Shortening long nominals
- d) Changing linguistically complex long sentences to shorter sentences
- e) Avoid using words with multiple meanings ("How many were *left*?)
- f) Simplify non-math vocabulary
- g) Reduce complex syntactic structures
- h) Shorten sentence length (give maximum lengths)
- i) Avoid comparative structures
- j) Avoid prepositional phrases
- k) Avoid conditional clauses
- l) Avoid relative clauses
- m) Avoid abstract or impersonal representations
- n) Unfamiliar contexts

Hanson et al. (1998) and Kopriva (2000) have a set of Plain Language guidelines

- 1) Same sentence structure should be retained throughout the assessment (Subject-verb-object)
- 2) Consistency in paragraph structures should be employed
- 3) Use present and active voice as much as possible
- 4) Rephrasing or rewording ideas and sentences should be kept to a minimum
- 5) Pronouns should be used as sparingly as possible
- 6) Use high-frequency words
- 7) Words with double meanings or colloquialisms should be omitted or defined in the text
 - a) See Math Register (Dale, Spanos, et al, 1988)

Kopriva (2000) has a set of guidelines about format and graphic organizers that will be useful in the TSNE work.

- 1) Visuals must be kept simple and to the point
- 2) Use a large enough font size
- 3) Reduce textual clutter by omitting use of item columns, limiting the number of items per page provide students a template to use ahead of time and get familiar with

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- 4) Lines or “boxes” which frame text or answer space in a judicious way

Visual issues

- 1) Visuals should be used to facilitate what is being asked or presented
- 2) Visuals should mirror or parallel the item statements or expectations
- 3) No unnecessary information should be placed in the visual to distract students from the item
- 4) Each major part of the visual should be represented in the visual
- 5) Simple text can and should be used in the visuals that correspond to important words in the item

Though this work has been focused on researching the best accommodations for English language learners (ELLs), we want to apply it and extend it for all students. Anecdotally we know that some students, for whom English is a first and only language, struggle with language issues as well, potentially contributing to an under representation of their true facility with math concepts, skills, and problem solving.

Lager and Petit (2003) – Streamlined language and Conserving the Mathematical Construct

Streamlined language includes the aforementioned Simplified and Plain Language rubrics (Abedi; Kopriva; Rivera, etc.), but goes beyond them by placing itself squarely in the interaction between the math construct, context, and format of an item. For example, in addition to the aforementioned streamlined language concerns related to Example I.9 (the 3 friends sharing the pie item) comes the following deliberate thinking around the choice of the word “classmate” and the necessary inclusion of the sentence “Your classmate says, ‘Each friend should pay \$4.50.’ ”

“Classmate” was chosen purposely to make the key claim “Each friend should pay \$4.50”, because he/she is outside the original group of 3 friends. This choice eliminates double references within the group, such as – “One of the friends says,” or “Each of us...”. Further, as the “Classmate” is making a false claim, it was deemed important to pick a subject who the student would readily accept as making a false claim. Other possible subjects who were considered were “Your teacher,” “Your mother (or father/sister/brother, etc.),” or “Your best friend.” Each option was rejected for a reason(s). Though every child has a teacher, we did not want to characterize the teacher as being mistaken about such a basic math concept. As every child does not have parents or siblings, those choices seemed inappropriate. Similarly, the “best friend” option was also nixed but also because we did not want another “friend” reference being confused with the 3 friends who buy the pie in the item. “Classmate” was chosen because every student has classmates and it is a gender-neutral option. This rationale parallels the choosing of “three friends” to replace Amy, Eric, and Kayla in the first sentence. (Note: Research should be done to test the hypothesis that using gender-neutral, abstract groups that can be personalized by students, such as friends and classmates, is better than using proper names of fictional protagonists.)

Also worth our attention is the necessary inclusion of the sentence “Your classmate says, ‘Each friend should pay \$4.50.’ ” At one point in the revision process, a rewritten

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version of this item excised that sentence, with the goal of eliminating unnecessary verbiage. However, without a human subject (the classmate) making that key conjecture for the student to consider, the item breaks down. Otherwise, a student would rightfully wonder where that \$4.50 amount came from if seeing it for the first time in the “Should each friend pay \$4.50?” query. When the item writer momentarily pushed too far, the construct “pushed back.” This is an example of how streamlining the language of an item interacts within the Conservation of the Mathematics Construct.

Bidders’ flexibility in regard to applying the concepts of *Conserving the Mathematical Construct*

The concepts and associated examples provided in this section are evolving. However, the basic principle of *Conserving the Mathematical* construct when using context and streamlining language is of utmost importance to the TSNE. Bidders should use the concepts and associated examples as a guide, and expect to work with the TSNE development teams to agree upon a set of guiding principles for *Conserving the Mathematical Construct*.

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H. Test Forms

Table 14: Sample Distribution of Common and Matrix Items across Item Types

	Estimated Time per Item	Number of Common Items	Number of Matrix Items	Total Testing Time (Minutes)	Number of Items Per Student	Number of Common Points Per Student
Selected Response - MC (1 point each)	1 Minute	34	4	38	38	34
Short Answer (1 points)	1 minute	6	2	8	8	6
Short Constructed Response (2 points)	2 minutes each	6	2	16	8	12
Medium Constructed Response	8 Minutes	4	2	48	6	12-16
	Totals	50	10	110	60	64-68

Bidders' flexibility in regard to test forms:

- 1) The assessment at each grade level should include approximately 50 items with the proposed distribution of item types. While there may be some flexibility in the distribution of item types, the number of MC should not exceed 35 MC in the common form, and the total testing time for common and matrixed items cannot exceed 2 hours. (See grade 3 exception in # 2 and # 3 below.)
- 2) Bidders can propose alternative distributions of items for the beginning grade 3 assessment that might have fewer, or no, medium to long constructed response items. More short answer and constructed response can be used in their place.
- 3) Bidders can propose alternative testing time at grade 3 like 2 – 45 sessions or 3 one-half hour sessions.

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I. Administration Considerations

Testing time – Each TSNE mathematics assessment will be designed to require no more than two (2) hours testing time divided equally across two test sessions for students in grades 5-8. For students in grades 3 and 4, the testing may be divided among three test sessions.

TSNE assessments should not be designed to be speed tests. A proficient student should be expected to complete the items in a session within the recommended time.

All TSNE assessments will be loosely timed. Students will be provided up to 25% additional time to complete each test session (except as specified in an accommodation). Where an accommodation specifies more additional time, the student(s) will be tested in a separate administrative location.

Accommodations – The TSNE supports the use of accommodations that do not threaten the validity of the assessment. Prior to the initial field test of items for the NSNE assessments, the NSNE states will review lists of acceptable accommodations in each state for the purpose of identifying a common set of allowable accommodations across the NSNE assessments. The TSNE states acknowledge that it may not be possible to reach agreement on a common set of allowable accommodations. When there are accommodations that specifically do threaten validity, these will be identified as well.

Calculator Use – The mathematics design team recommends the following in regards to calculator use:

- Grades 3 and 4: No calculators for any aspect of the assessment.
- Grades 5 – 8: One session, or a segment of a session, should be calculator free. Items during the calculator free session should not be calculator dependent and should provide information about fluency with number operations, but should not be “naked” computation.

Use of classroom materials – The guidelines for the use of classroom materials during the TSNE grade level assessments have not been established at this point.

Providing manipulatives - If specific tools or manipulatives are required in an item, they will be provided to students as a part of the testing materials. The TSNE partners will identify GLEs that will require materials, and the materials necessary.

Formula Sheet - The guidelines for the inclusion of a formula sheet during the TSNE grade level assessments have not been established at this point.

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Special Considerations in Test Design and Administration at Grade Three:

There has been concern among the TSNE partners about the administration of the beginning grade 3 mathematics assessments. The concerns have ranged from the attention span to communication (writing and reading) independence of students of this age level. After considerable discussion the TSNE partners have arrived at the following agreements.

- Students will read and respond to questions in the same test booklet. They will not be asked to bubble-in items on a separate scan sheet;
- Students will read items and directions independently (as contrasted with having an assessment that would include oral reading of items by an adult);
- Constructed response items and associated rubrics should allow for multiple forms of expression; pictures, text, or mathematical language;
- A thorough set of accommodations for those students who may need additional support (reading items aloud) should be developed;
- When appropriate, problems should be presented (either MC or CR) using pictures (e.g., base 10 blocks) to minimize the effects of reading ability;
- Review items for “grade-appropriate” and clear language; apply guidelines for Conserving the mathematical construct; and
- Divide the total testing time into three testing sessions at grade 3.

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Table 15: Summative Table of Rationales and Bidders Flexibility

Components of Test Specifications	Rationale	Bidder Flexibility
Content Strands and Reporting Categories	The content strands were identified by the TSNE mathematics content team as the organizational structure for the TSNE GLEs and the reporting categories.	<ul style="list-style-type: none"> The content strands identified will be used as sub-reporting areas. Raw scores or percent of points earned shall be reported for any content strand that has 10 or more points. Bidders should provide methods of reporting the raw data for the content strands, and any other diagnostic information that can be derived from the assessment.
Distribution of Emphasis (DoE) across content strands and GLEs within content strands	The Distribution of Emphasis was established to provide a distribution of items across a form that would result in stable forms across years that reflect recommended assessment emphasis in relationship to the TSNE GLEs and accounts for sampling.	<ul style="list-style-type: none"> Some aspect of each GLE should be assessed every year. The distribution of items across the content strands should be consistent (to the degree possible +/- 5%) with the recommended TSNE Distribution of Emphasis. The TSNE partnership would not expect that the number of items across GLEs be rigidly adhered to from year to year, but does expect that the relative emphasis be attended to in sampling. Because the TSNE partner is interested in some GLEs being assessed deeper with more items, TSNE would not accept a GLE designated as low emphasis receiving high emphasis, vice versa. The relative balance as reflected by the number of points possible will potentially vary from the Distribution of Emphasis across and within content strands as items types with their different point values are distributed across the GLEs. Contractors should provide strategies that consider the maintenance of Distribution of Emphasis for both items and points.
Depth of Knowledge “ceilings”	Including items that assess the cognitive range of a GLE is designed to avoid two potentially negative impacts on the assessment. 1) The assessment as a whole would be too difficult; and 2) important information about student learning along the achievement continuum would be lost. (<i>Vermont Revised Draft Test Specifications, June 2002</i>)	<ul style="list-style-type: none"> Distribution of Depth of Knowledge across the assessment should stay within the general guidelines of provided by the TSNE partners. The general alignment protocol for this aspect is that a GLE should not be assessed above its “ceiling”, and to the extent possible at the “ceiling” and at least one level below the “ceiling.” Bidders should include strategies to address the distributions across the levels. Bidders should indicate any concerns or limitations that may arise from specification in # 2 above. Bidders can provide alternative strategies to address this specification that assures that the interaction of content with cognitive demand in incorporated into the design. Bidders should be prepared to work with the TSNE content teams to solidify the TSNE Depth of Knowledge descriptors.

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Components of Test Specifications	Rationale	Bidder Flexibility
Interaction of Content and DoK	Provide a conceptual model to consider in development	<ul style="list-style-type: none"> This element is a conceptual model. The TSNE does not expect contractors to fill all the cells in the models above, but does expect contractors to avoid scenarios where the hardest content is assessed at the highest level, and easier content at the lowest level.
Conjunctions – “ands”, “ors”	The “ands” and “ors” were established for test developers to guide sampling as it reflects the prioritization that occurred throughout the TSNE GLE development.	<ul style="list-style-type: none"> Every GLE should be assessed by at least 1 item every year. To the extent possible elements of GLEs connected with “ands” should be assessed every year. Elements of the GLE connected with “or” can be rotated across years. Contractors should develop items that provide the opportunity for students to select their own strategies or methods of communication throughout the assessment. Particular attention should be paid to GLEs where elements are connected by “or^{sc}.”
Item Types	To provide multiple item formats that include opportunities to explain reasoning and show work when solving problems.	<ul style="list-style-type: none"> The assessment at each grade level must include a distribution of all four-item types. (See grade 3 exception in # 6 below.) The total number of short answer and short constructed response items should be about 12; not necessarily 6 of each type. The assessment at each grade level should include approximately 50 items to maintain adequate sampling. (See grade 3 exception in # 6 below.) The assessment at each grade level should include 4 medium-long constructed response items. The current thinking is one for each content strand. (See grade 3 exception in # 6 below.) The testing period cannot exceed 2 – one-hour sessions. Contractors can propose alternative distributions of items for the beginning grade 3 assessment that might have fewer, or no, medium to long constructed response items. More short answer and constructed response may be used in their place. Potential bidders should provide the TSNE partners with examples of rubrics to assess each of these item types. Each constructed response item will include an item-specific analytic scoring guide that includes the following: detailed descriptions of required performance at each score point.
Conserving the Mathematics Construct	To provide access to mathematics expected in items to the greatest number of students by streamlining language, using rich contexts, when appropriate, without compromising the mathematical construct being assessed.	<ul style="list-style-type: none"> The concepts and associated examples provided in this section are evolving. However, the basic principle of <i>Conserving the Mathematical</i> construct when using context and streamlining language is of utmost importance to the TSNE. Bidders should use the concepts and associated examples as a guide, and expect to work with the TSNE development teams to agree upon a set of guiding principles for <i>Conserving the Mathematical Construct</i>.

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Components of Test Specifications	Rationale	Bidder Flexibility
Test Forms	A common and matrix form is used to assure that the assessment is reliable and valid, is sustainable, and can be equated from year to year.	<ul style="list-style-type: none">• The assessment at each grade level should include approximately 50 items with the proposed distribution of item types. While there may be some flexibility in the distribution of item types, the number of MC should not exceed 35 MC in the common form, and the total testing time for common and matrixed items cannot exceed 2 hours. (See grade 3 exception in # 2 and # 3 below.)• Bidders can propose alternative distributions of items for the beginning grade 3 assessment that might have fewer, or no, medium to long constructed response items. In their place may be more short answer or short constructed response items.• Bidders can propose alternative testing time at grade 3 like 2 – 45 sessions or 3 one-half hour sessions.

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Appendix A: The Tri-State New England GLEs (TSNE GLEs)

Purpose of TSNE GLEs: TSNE GLEs are specified for the development of a common, large-scale, state level assessment. Partner states have or may include additional GLEs for use within each state for local assessment purposes.

Definition of a TSNE GLE:

A TSNE GLE is a stated objective that is aligned with VT, NH, RI, and ME standards, by grade. A NEC GLE differentiates performance on concepts, skills, or content knowledge between adjacent grade levels, and as a set, leads to focused, coherent, and developmentally appropriate instruction without narrowing the curriculum.

Criteria for the Development of TSNE GLEs

- 1) TSNE GLEs **must** relate to the RI, ME, NH, and VT standards.
- 2) TSNE GLEs should maintain a balance between a generalizable skill, concept, or piece of knowledge, **and** enough specificity to differentiate skill, concept, or knowledge between adjacent grades, to make it clear to teachers what is to be taught and learned, ***without being so specific that it narrows the curriculum.***
- 3) TSNE GLEs should explicitly indicate cognitive demand (interaction of content and process). There should be a mix of cognitive demands at all grade levels, not an assumption that students in lower grades do less cognitively demanding work. (E.g., Routine skill/procedure, conceptual problem or question, multiple-step problems, problem solving, analysis, reasoning, etc.)
- 4) TSNE GLEs should be specific and clear enough to know how it will be assessed.
- 5) TSNE GLEs should contain language that describes expected performance so that a student's performance in relation to the NEC GLE can be validly assessed for state assessment purposes.

Not assessable – E.g., “Develops understanding of plot..” or “Begins to use..”.

Assessable – E.g., Demonstrates understanding of plot by identifying and describing characters, setting, problem/solution, and plot.

Note: Test specifications might indicate the upper ceiling of that expectation for state assessment purposes or explicit strategies for assessing the TSNE GLEs. Demonstrate understanding of plot by: a) identifying and describing characters; b) given an incomplete story sequence, extend the story with a coherent “next step”; or c) given a definition of plot development, provide an example.]

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TSNE Criteria for the Development of a *SET* of TSNE GLEs

1. The set of TSNE GLEs should be of comparable grain size.
2. Concepts, skills, and knowledge should be differentiated between adjacent grade levels.
3. The set of TSNE GLEs within a discipline and content standard reflects the relative importance as defined by the Balance of Representation.
4. The set of TSNE GLEs should promote coherent, focused, developmentally appropriate instruction, as opposed to isolated instruction *just* on topics, facts, or individual skills that need to be covered.
5. The set of TSNE GLEs at a given grade level (assuming prior learning) should be reasonable to adequately learn within a school year, and still allow for learning additional state and local expectations.
6. The set of TSNE GLEs should be constructed as a continuum of learning. Success in one grade should be a good predictor of success in the following year.
7. Success on TSNE GLEs across multiple years should be a good predictor of performance at the national benchmark years. (i.e., NAEP).

Questions to guide prioritization:

- 1) Is the concept or skill part of a big idea in the discipline? (E.g., proportionality)
- 2) Is the success on the concept or skill in a given grade essential for success in mathematics in subsequent grades?
- 3) Should the concept or skill be assessed at an earlier grade because success at that earlier grade is important for success at the given grade?
- 4) Is the concept or skill assessed adequately at an earlier grade?
- 5) Should the concept or skill be assessed at a later grade for state assessment purposes?
- 6) Is the concept or skill subsumed in other GLEs at that grade level? (E.g., Is the skill of applying the conventions of order of operations subsumed in the use of formulas, and when solving multistep linear equations at grade 8?)
- 7) Are concepts or skills important for success in other disciplines in given grades or subsequent grades?
- 8) Is the concept or skill better assessed in the classroom? (e.g., Mentally adds and subtracts...)

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